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Low Endurance Fatigue in Metals and Polymers

Part 2 *Fatigue Damage*

B. TOMKINS*, W. D. BIGGS
Engineering Laboratory, University of Cambridge, UK

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The fatigue behaviour of commercially pure aluminium and of nylon under sequentially varying strain amplitudes is compared with a damage law of the type suggested by Miner. Aluminium obeys such a law for both cyclic and uniaxial prestrains but the behaviour of nylon is significantly affected by microcracking, which produces a marked effect of loading sequence.

1. Introduction

The preceding paper [1] examined the stress/strain relationship in aluminium, nylon and an epoxy resin subjected to low life (10^4 cycles) cyclic loading. In practice the fatigue loading of a structure is a random process and, in order that test data may be of value in design, it is often desirable to establish a criterion of damage at various levels of stress and strain. Miner [2] suggested that, under stress-cycling conditions

$$D = \sum_{i=1}^{n} \frac{N_i}{N_{fi}} = 1 \text{ (at failure)} \qquad (1)$$

(Here, N_i represents the number of cycles under a stress amplitude $\Delta\sigma_i$, and N_{fi} is the number of cycles to failure under this regime, if one starts with unstrained material). Later investigators found that the sequence of loading affected the predictions of equation 1 and Topper [3] showed that, since Miner's assumptions were based on an absorbed energy criterion for failure, strain-cycling should satisfy equation 1 better than stress-cycling. Thus, for mild steel [4] it was found that a modified form of equation 1 was satisfactory, viz.

$$D = \sum_{i=1}^{n} \frac{N_i \Delta\epsilon_{pi}^{1/\alpha}}{C^{1/\alpha}} = 1 \text{ (at failure)} \qquad (2)$$

Here α and C are the Coffin-Manson parameters (see preceding paper). In the present tests, aluminium and nylon specimens were cycled at various strain ranges in both ascending and descending sequence and their behaviour was assessed in terms of equation 2.

2. Materials etc

The materials and test procedure were as described in the previous paper, viz. commercial purity aluminium, nylon rod stock; testing speed was 300 c/m.

3. Aluminium

Fig. 1 shows test results from sequential loading in relation to the Coffin-Manson failure line for the material. A horizontal line is drawn at the first strain range $\Delta\epsilon_{p1}$ for the period of test (N_1); a line from the point so defined is then drawn parallel to the failure line to the second strain range $\Delta\epsilon_{p2}$. The number of cycles remaining at $\Delta\epsilon_{p2}$ is the residual life (N_2) to failure at a total life N_f.

From equation 2 the values of D for a descending sequence were 1.10 and 1.02, and for an ascending sequence 1.05 with an average of $D = 1.06$. We conclude then that there is no sequential effect and the damage criterion is obeyed at low endurances.

Various authors have observed that, in damage

*Now at UKAEA (Reactor Group), Springfields, Lancs, UK.

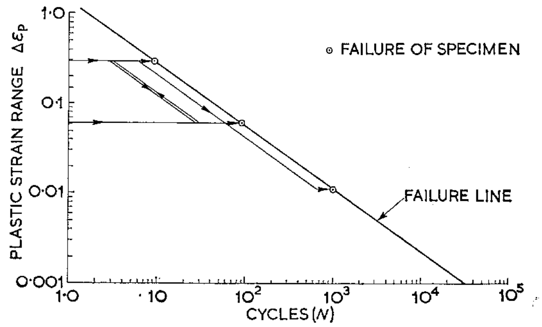

Figure 1 CP aluminium – effect of sequential loading on failure. Failure line from fig. 11 of [1].

tests, softening or further hardening occurs at the second stage depending on whether this was lower or higher respectively than the first stage. In particular Dugdale [5] showed that, for an aluminium alloy, copper and mild steel, a limiting level of $\Delta\sigma$ would eventually be reached for a given $\Delta\epsilon$, i.e. that prior strain history had no effect. Fig. 2 is generally in accord with this finding; the final steady stress range approached that for the annealed material. The softening curves were found to be initially approximately exponential and, thereafter, linear for most of the life. This compares with the linear hardening of initially annealed material.

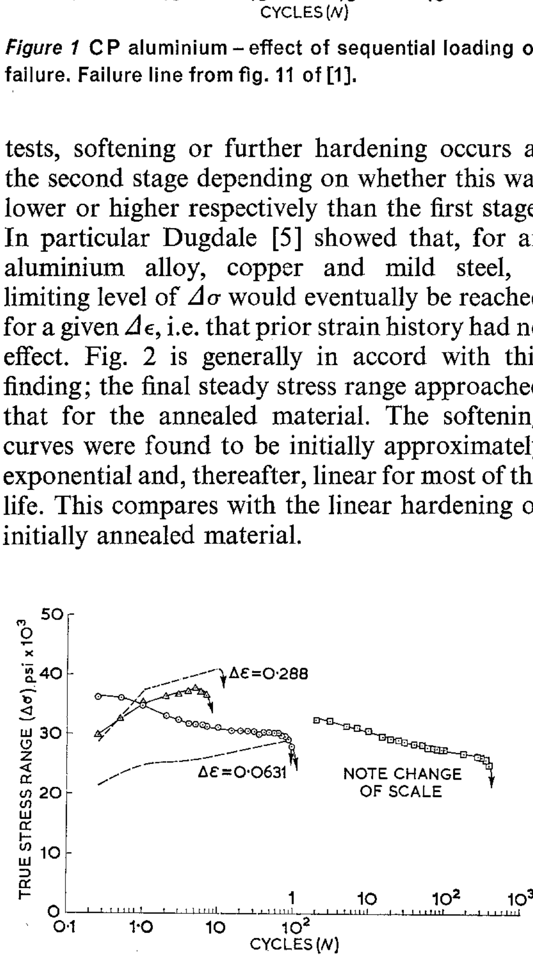

Figure 2 CP aluminium – stress range versus cycles for second stage cyclic tests. Circles $\Delta\epsilon_1 = 0.288$, $N_1 = 3$, $\Delta\epsilon_2 = 0.0631$; triangles $\Delta\epsilon_1 = 0.0631$, $N_1 = 30$, $\Delta\epsilon_2 = 0.288$; squares $\Delta\epsilon_1 = 0.288$, $N_1 = 6$, $\Delta\epsilon_2 = 0.0134$. The dashed curves refer to the annealed material at the values of $\Delta\epsilon$ given.

Biggs and Topper [4] found that, for mild steel, an initial static prestrain could be expressed in terms of an equivalent cyclic damage; this was also checked in the present tests. The total initial prestrain composed of the uniaxial strain ϵ_p plus the first cyclic straining $\Delta\epsilon_p/2$ is expressed as an equivalent strain range $\Delta\epsilon_p = 2\epsilon_p$ at $N = 0.25$ cycles. The "prestrain damage" D_p can then be expressed as the ratio of $N_{0.25} : N_f$ (from fig. 11 of the previous paper [1]), for the plastic strain range $\Delta\epsilon_p$. Fig. 3 shows that, at the lower strain ranges, the failure points lie on the cyclic failure line. The deviation at the higher strain range suggests that these strains are more damaging than predicted.

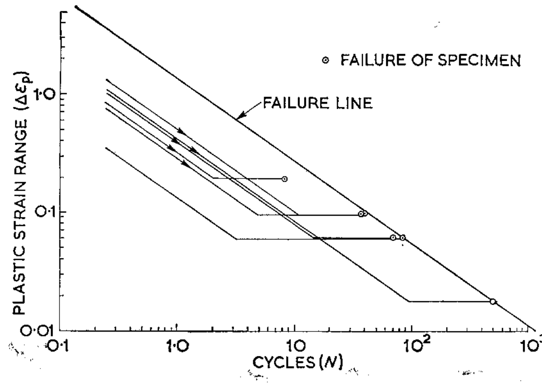

Figure 3 CP aluminium – effect of static prestrain. Failure line from fig. 11 of [1].

The hardening and softening behaviour for two different static prestrains is shown in fig. 4. The variation in $\Delta\sigma$ with cycles is linear over most of the life in all cases. For a given strain range all curves tend toward a limiting value of $\Delta\sigma$ which is reached by the initially annealed specimen (as for the two-stage cyclic tests described above). It should be noted that, for aluminium, the limiting stress range is that maximum which is reached at the *end* of the test on annealed material. With other materials which do not continuously harden it is the steady stress range reached *after the initial hardening period* which is approached by the cycled material. It can be seen from fig. 4 that the limiting stress range is not attained by the prestrained specimen after the initial drop in stress, i.e. at the start of the linear region.

4. Nylon

Five tests were performed using both descending and ascending sequences – the strain ranges chosen gave endurances of 125 and 17×10^3 cycles on annealed material. As the true plastic strain range could not be assessed the *total* strain

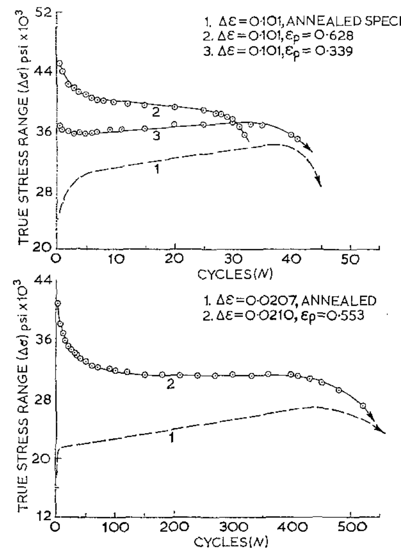

Figure 4 Nylon – stress range versus cycles for cyclic tests after tensile prestrain (ϵ_p).

range replaces the plastic strain range in equation 2. The results are summarised in table I. For tests in descending sequence, $D = 1.40$ and 0.95, indicating that the damage criterion is only approximately obeyed. The tests in ascending sequence show a different behaviour: for specimen NY 26 and NY 27 the number of cycles at the (high) second strain was much smaller than would be predicted, whereas for NY 28 it was almost equal to the endurance of an annealed specimen at this range.

Examination of the fracture surfaces showed that, in the descending sequence tests, the cracks were initiated at the surface under the high strain and propagated under the low strain. In the ascending sequence tests NY 26 and NY 27 the initial low strain nucleated internal cracks which clearly led to rapid structural breakdown under the cycles of higher strain. A similar disastrous effect of prior low strain cycling has been reported by Wood *et al* [6], who found internal pore and microcrack development in brass. No evidence of internal microcracking could be discerned on the fracture surface of NY 28; the fracture surface was identical with that of an annealed sample. These results suggest that the main crack is nucleated between 20 and 40% of the life at a strain range giving a life of 17×10^3 cycles. This is greater than the estimate of 10% for metals given by Thompson [7] and may be a consequence of the fact that nylon has a much more limited capacity for the cross-slip which is necessary for rapid crack initiation in metals.

The variation of stress range with cycles is shown in fig. 5. Curves are shown for the cycles at the second strain range and are compared with those of annealed material at these ranges. The curves for the descending sequence tests both lie below the "annealed" curve; this is consistent with the softening which could have occurred during the first strain cycles. (The slight increase over the first twenty to thirty cycles in one case is evident of some stress-recovery.) The curves for the ascending sequence tests deviate little from that for the annealed material indicating that the microstructural deterioration which occurred had little effect on the load bearing capacity of the material.

5. Discussion

The existence of a linear damage rule for applied cyclic strains of sequentially varying amplitude applies to aluminium. The Coffin-Manson law is basically a crack propagation criterion and is indicative of a direct dependence of crack growth rate on applied strain range. A linear damage criterion of strain range suggests, therefore, that the crack growth rate is independent of the strain history. When the cyclic strain range is varied, the crack growth rate adjusts to

TABLE I

	Specimen	$\Delta\epsilon_1$	N_1	N_{1f}	N_1/N_{1f}	$\Delta\epsilon_2$	N_2	N_{2f}	N_2/N_{2f}	D
High strain followed by low strain	NY 35	0.460	100	125	0.80	0.143	10 200	17 000	0.60	1.40
	NY 31	0.460	60	125	0.48	0.143	7900	17 000	0.47	0.95
Low strain followed by high strain	NY 26	0.143	12 000	17 000	0.71	0.460	8.25	125	0.07	0.78
	NY 27	0.143	7000	17 000	0.41	0.460	7.25	125	0.06	0.47
	NY 28	0.143	3000	17 000	0.18	0.460	177*	125	1.41	1.59

where $N_{if} = \left(\frac{C_N}{\Delta\epsilon_i}\right)^{1/\alpha_N}$

*This endurance is 41% greater than the predicted endurance of 125 cycles at $\Delta\epsilon = 0.460$. However, an annealed specimen cycled to failure at this strain range had an endurance of 185 cycles.

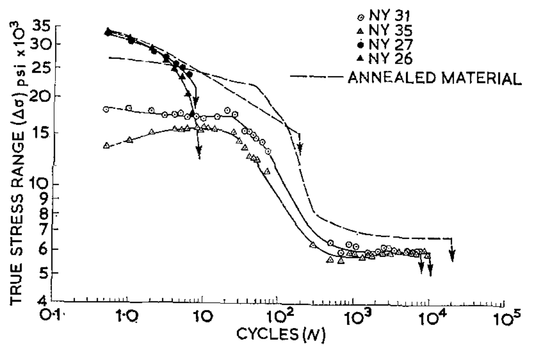

Figure 5 Nylon – stress range versus cycles for second stage cyclic tests. NY 31 $\Delta\epsilon_1 = 0.460, N_1 = 60, \Delta\epsilon_2 = 0.143$; NY 35 $\Delta\epsilon_1 = 0.460, N_1 = 100, \Delta\epsilon_2 = 0.143$; NY 26 $\Delta\epsilon_1 = 0.143, N_1 = 12 \times 10^3, \Delta\epsilon_2 = 0.460$; NY 27 $\Delta\epsilon_1 = 0.143, N_1 = 7 \times 10^3, \Delta\epsilon_2 = 0.460$.

that strain range at the crack length then attained. No sequential effect would be expected on this basis.

The reason for a linear damaging effect of static prestrain in aluminium is not clear, though it is possible that the effect is approximately linear only at the lower strain ranges used. At higher strain ranges the damage caused by static prestrain is greater than that predicted from a linear law. This may be associated with crack initiation, possibly simply by the increased surface roughening produced.

The cyclic softening and hardening of previously strained aluminium was found to be initially exponential with cycles and, thereafter, linear with cycles over most of the life. The explanation for this must presumably be sought in the rearrangement of dislocations under cyclic straining and it is suggested that the initial, exponential stage proably reflects disruption of the prestrained structure (cyclic or static) into a dislocation substructure appropriate to the particular cyclic strain range. The reason for the existence of linear softening is not at all clear, though the eventual attainment of a particular stress response for a given strain is presumably associated with the substructure developed at that strain range. Further studies are in hand to investigate substructure development and its relation to applied strain range.

Although nylon also obeys a failure criterion of the Coffin-Manson type the linear damage rule applies only to descending strains and then only approximately. In the case of ascending strains the crack growth rate is significantly affected by the growth of internal cracks.

The few results on nylon damage suggest that a process of structural shakedown occurs; this leads to a unique stress response to a given strain range independent of strain history.

6. Conclusions

(i) In the low endurance range, damage due to strain-cycling in aluminium can be assessed by means of a modified Miner law based on a plastic strain failure criterion. Uniaxial prestrain damage can be assessed in the same way at intermediate and low prestrain.

(ii) Nylon, although obeying a strain criterion of failure, only satisfied the damage criterion for a descending sequence of strains. The damaging effect of ascending strains is greater than predicted due to the formation of internal microcracks.

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Appendix

N = Number of strain cycles at a given time
N_f = Value of N at failure.
σ = True tensile stress.
$\Delta\sigma$ = True stress range for a strain cycled specimen.
$\Delta\sigma_h$ = Value of $\Delta\sigma$ at half the life of the specimen.
ϵ = True tensile strain.
$\Delta\epsilon$ = Total true strain range.
$\Delta\epsilon_p$ = True plastic strain range (= the breadth of the hysteresis loop at $\sigma = 0$).
$\Delta\epsilon_d$ = True diametral strain range.
E = Young's modulus.
γ = Linear strain hardening rate when tested at a particular value of $\Delta\epsilon_p$.
D = Damage due to cycling.
D_p = Damage due to prestrain.
ϵ_p = Prestrain.
C, K, K_1, α, β are constants.

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